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Brane Inflation from Mirage Cosmology.

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Abstract

We study the cosmological evolution of a D3-brane universe in a type 0 string background. We follow the brane-universe along the radial coordinate of the background and we calculate the energy density which is induced on the brane because of its motion in the bulk. We find that for some typical values of the parameters the brane-universe has an inflationary phase.

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1. Introduction

There has been much recent interest in the idea that our universe may be a brane embedded in some higher dimensional space [1]. It has been shown that the hierarchy problem can be solved if the higher dimensional Planck scale is low and the extra dimensions large [2]. Randall and Sundrum [3] proposed a solution of the hierarchy problem without the need for large extra dimensions but instead through curved five-dimensional spacetime AdS_5 that generates an exponential suppression of scales.

This idea of a brane-universe can naturally be applied to string theory. In this context, the Standard Model gauge bosons as well as charged matter arise as fluctuations of the D-branes. The universe is living on a collection of coincident branes, while gravity and other universal interactions is living in the bulk space [4].

This new concept of brane-universe naturally leads to a new approach to cosmology. Any cosmological evolution like inflation has to take place on the brane while gravity acts globally on the whole space. In the literature there are a lot of cosmological models which study the cosmological evolution of our universe. In most of these models the spacetime is five-dimensional, where the fifth dimension is the radial dimension of an AdS_5 space. The effective Einstein equations on the brane are then solved taking under consideration the matter on the brane [5]-[8].

Another approach to cosmological evolution of our brane-universe is to consider the motion of the brane in higher dimensional spacetimes. In [7] the motion of a domain wall (brane) in such a space was studied. The Israel matching conditions were used to relate the bulk to the domain wall (brane) metric, and some interesting cosmological solutions were found. In [8] a universe three-brane is considered in motion in ten-dimensional space in the presence of a gravitational field of other branes. It was shown that this motion in ambient space induces cosmological expansion (or contraction) on our universe, simulating various kinds of matter.

In this direction we have studied [9], the motion of a three-brane in the background of type 0 string theory . It was shown that the motion of the brane on this specific background, with constant values of dilaton and tachyon fields, induces a cosmological evolution which for some range of the parameters has an inflationary phase. In [10], using similar technics the cosmological evolution of the three-brane in the background of type IIB string theory was considered.

Type 0 string theories [11] are interesting because of their connection [12] to four-dimensional $SU(N)$ gauge theory. Asymptotic solutions of the theory were constructed in [11, 14]. At large radial coordinate the tachyon is constant and one finds a metric of the form $AdS_5 \times S^5$ with vanishing coupling which was interpreted as a UV fixed point. The solution exhibits a logarithmic running in qualitative agreement with the asymptotic freedom property of the field theory. At small radial coordinate the tachyon vanishes and one finds again a solution of the form $AdS_5 \times S^5$ with infinite coupling, which was interpreted as a strong coupling IR fixed point.

From the holographic principle and the AdS/CFT correspondence, this renormalization group flow of type 0 string theory, can be understood to correspond to moving the brane a finite distance in the bulk. Motivated by this behaviour of type 0 theory, we will discuss in this talk the cosmological evolution of the brane-universe as the brane moves from the UV to the IR fixed point.

We calculate the effective energy density which is induced on the brane because of its motion in the particular background of a type 0 string. Using the approximate solutions of [11, 14], we find that for large values of the radial coordinate r , in the UV region, the effective energy density takes a constant value, which means that the universe has an inflationary period.

2. Brane-Universe

In [8] it was considered a D-brane moving in a generic static, spherically symmetric background. As the brane moves in a geodesic, the induced world-volume metric becomes a function of time, so there is a cosmological evolution from the brane point of view. The metric of a D3-brane is parametrized as

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5 \quad (2.1)$$

and there is also a dilaton field Φ as well as a RR background $C(r) = C_{0\dots 3}(r)$ with a self-dual field strength. The action on the brane is given by

$$\begin{aligned} S = & T_3 \int d^4\xi e^{-\Phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta} - B_{\alpha\beta})} \\ & + T_3 \int d^4\xi \hat{C}_4 + \text{anomaly terms} \end{aligned} \quad (2.2)$$

The induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu \partial x^\nu}{\partial \xi^\alpha \partial \xi^\beta} \quad (2.3)$$

with similar expressions for $F_{\alpha\beta}$ and $B_{\alpha\beta}$. In the static gauge, using (2.3) we can calculate the bosonic part of the brane Lagrangian which reads

$$L = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r) \quad (2.4)$$

where $h_{ij}d\varphi^i d\varphi^j$ is the line element of the unit five-sphere, and

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\Phi}, B(r) = g^3(r)g_{rr}(r)e^{-2\Phi}, D(r) = g^3(r)g_S(r)e^{-2\Phi} \quad (2.5)$$

Demanding conservation of energy E and of total angular momentum ℓ^2 on the brane, the induced four-dimensional metric on the brane is

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2 \quad (2.6)$$

with η the cosmic time which is defined by

$$d\eta = \frac{|g_{00}|g^{\frac{3}{2}}e^{-\Phi}}{|C+E|}dt \quad (2.7)$$

This equation is the standard form of a flat expanding universe. If we define the scale factor as $\alpha^2 = g$ then we can calculate the Hubble constant $H = \frac{\dot{\alpha}}{\alpha}$, where dot stands for derivative with respect to cosmic time. Then we can interpret the quantity $(\frac{\dot{\alpha}}{\alpha})^2$ as an effective matter density on the brane with the result

$$\frac{8\pi}{3}\rho_{eff} = \frac{(C+E)^2 g_S e^{2\Phi} - |g_{00}|(g_S g^3 + \ell^2 e^{2\Phi})}{4|g_{00}|g_{rr}g_S g^3} \left(\frac{g'}{g}\right)^2 \quad (2.8)$$

Therefore the motion of a D3-brane on a general spherically symmetric background had induced on the brane a matter density. As it is obvious from the above relation, the specific form of the background will determine the cosmological evolution on the brane.

3. Type 0 string background

The action of the type 0 string is given by [11]

$$\begin{aligned} S_{10} = & \int d^{10}x \sqrt{-g} \left[e^{-2\Phi} \left(R + 4(\partial_\mu \Phi)^2 - \frac{1}{4}(\partial_\mu T)^2 - \frac{1}{4}m^2 T^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \right) \right. \\ & \left. - \frac{1}{4} \left(1 + T + \frac{T^2}{2} \right) |F_5|^2 \right] \end{aligned} \quad (3.1)$$

The tachyon is coupled to the RR field through the function

$$f(T) = 1 + T + \frac{1}{2}T^2 \quad (3.2)$$

In the background where the tachyon field acquires vacuum expectation value $T_{vac} = T_0 = -1$, the tachyon function (3.2) takes the value $f(T_0) = \frac{1}{2}$ which guarantee the stability of the theory [11].

Technically it is easier to solve the equations of motion resulting from the above action, if we go to new variables. One can then introduce new parameter ρ through the relation

$$\rho = \frac{e^{2\Phi_0}}{4r^4} \quad (3.3)$$

and the fields ξ and η from the relations

$$g = e^{\frac{\Phi-\xi}{2}}, g_s = e^{\frac{\Phi+\xi}{2}-\eta} \quad (3.4)$$

Then (2.1) takes the form,

$$ds^2 = -e^{\frac{\Phi-\xi}{2}} dt^2 + e^{\frac{\Phi-\xi}{2}} d\vec{x}^2 + e^{\frac{\Phi+\xi}{2}-5\eta} d\rho^2 + e^{\frac{\Phi+\xi}{2}-\eta} d\Omega_5^2 \quad (3.5)$$

With this form of the metric, the action (3.1) can be described by the following Toda-like mechanical system (dot denotes ρ -derivative)

$$S = \int d\rho \left[\frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 + \frac{\dot{T}^2}{4} - 5\dot{\eta}^2 - V(\Phi, \xi, T, \eta) \right] \quad (3.6)$$

with the potential $V(\Phi, \xi, T, \eta)$ given by

$$V(\Phi, \xi, T, \eta) = g(T) e^{\frac{1}{2}\Phi + \frac{1}{2}\xi - 5\eta} + 20e^{-4\eta} - Q^2 f^{-1}(T) e^{-2\xi} \quad (3.7)$$

If the tachyon field takes its vacuum value and the dilaton field a constant value $\Phi = \Phi_0$ one can find the electrically charged three-brane

$$g_{00} = -H^{-\frac{1}{2}}, g(r) = H^{-\frac{1}{2}}, g_S(r) = H^{\frac{1}{2}} r^2, g_{rr}(r) = H^{\frac{1}{2}}, H = 1 + \frac{e^{\Phi_0} Q}{2r^4} \quad (3.8)$$

if the following ansatz for the RR field is used

$$C_{0123} = A(r), F_{0123r} = A'(r) \quad (3.9)$$

If T and Φ are functions of the coordinate r , then approximate solutions exist [11, 14]. These solutions are valid for large (UV) and small (IR) values of the radial coordinate. From the action (3.6) we can derive the following equations of motion [14]

$$\ddot{\xi} + \frac{1}{2} g(T) e^{\frac{1}{2}\Phi + \frac{1}{2}\xi - 5\eta} + 2 \frac{Q^2}{f(T)} e^{-2\xi} = 0 \quad (3.10)$$

$$\ddot{\eta} + \frac{1}{2} g(T) e^{\frac{1}{2}\Phi + \frac{1}{2}\xi - 5\eta} + 8e^{-4\eta} = 0 \quad (3.11)$$

$$\ddot{\Phi} + \frac{1}{2} g(T) e^{\frac{1}{2}\Phi + \frac{1}{2}\xi - 5\eta} = 0 \quad (3.12)$$

$$\ddot{T} + 2g'(T)e^{\frac{1}{2}\Phi + \frac{1}{2}\xi - 5\eta} + 2\frac{Q^2 f'(T)}{f^2(T)}e^{-2\xi} = 0 \quad (3.13)$$

where $g(T)$ is the bare tachyon potential,

$$g(T) = \frac{1}{2}T^2 - \lambda T^4 \quad (3.14)$$

and λ is a parameter. Defining a new variable $\rho = u^{-4}$, in the UV for $u \longrightarrow \infty$, or $\rho \longrightarrow 0$, we can solve the equations of motion (3.10)-(3.13) to the next to leading order and find [14, 11]

$$T = T_0 - 4\frac{g'(T_0)}{g(T_0)}\frac{1}{\log\rho} + O\left(\frac{\log(-\log\rho)}{\log^2\rho}\right) \quad (3.15)$$

$$\Phi = -2\log(C_0\log\rho) - \left(7 + 8\left(\frac{g'(T_0)}{g(T_0)}\right)^2\right)\left(\frac{\log(-\log\rho)}{\log\rho}\right) + \frac{B}{\log\rho} + O\left(\frac{\log^2(-\log\rho)}{\log^2\rho}\right) \quad (3.16)$$

$$\xi = \log\left(\sqrt{2f^{-1}(T_0)Q\rho}\right) - \frac{1}{\log\rho} + O\left(\frac{\log(-\log\rho)}{\log^2\rho}\right) \quad (3.17)$$

$$\eta = \frac{1}{2}\log(4\rho) - \frac{1}{\log\rho} + O\left(\frac{\log(-\log\rho)}{\log^2\rho}\right) \quad (3.18)$$

where $C_0 = -\frac{4C_2^5}{g(T_0)\sqrt{C_1}}$ and

$$C_1 = \frac{2Q}{\sqrt{2f(T_0)}}\left(1 + \frac{1}{4\log\frac{u}{u_0}}\right), C_2 = 2\left(1 + \frac{1}{4\log\frac{u}{u_0}}\right) \quad (3.19)$$

The above solutions show that at the UV point the tachyon takes a constant value and if we calculate the effective coupling $e^{\frac{1}{2}\Phi}$ we will see that it goes to zero for large u .

For $u \longrightarrow 0$ or for large ρ the approximate solutions are

$$T = -\frac{16}{\log\rho} - \frac{8}{\log^2\rho}(9\log\log\rho - 3) + O\left(\frac{\log^2\log\rho}{\log^3\rho}\right) \quad (3.20)$$

$$\Phi = -\frac{1}{2}\log(2Q^2) + 2\log\log\rho - \frac{1}{\log\rho}9\log\log\rho + O\left(\frac{\log\log\rho}{\log^2\rho}\right) \quad (3.21)$$

$$\xi = \frac{1}{2}\log(2Q^2) + \log\rho + \frac{9}{\log\rho} + \frac{9}{2\log^2\rho}(9\log\log\rho - \frac{20}{9}) + O\left(\frac{\log^2\log\rho}{\log^3\rho}\right) \quad (3.22)$$

$$\eta = \log 2 + \frac{1}{2}\log\rho + \frac{1}{\log\rho} + \frac{1}{2\log^2\rho}(9\log\log\rho - 2) + O\left(\frac{\log^2\log\rho}{\log^3\rho}\right) \quad (3.23)$$

The tachyon field in the IR point goes to zero while the effective coupling gets infinite. It is important to observe that the approximate solutions in the UV (3.15)-(3.18) and in

the IR (3.20)-(3.23) of [11] are related by $y \longrightarrow -y$, suggesting that they can be smoothly connected into a full interpolating solution. As we mention above, the tachyon field starts at $T=-1$ at $\rho = 0$ in the UV, and grows according to (3.15), then enters an oscillating regime and finally relaxes to zero according to (3.20), when $\rho = \infty$ in IR. This guarantees that $T^2 e^{\frac{1}{2}\Phi}$ becomes small which leads the metric in the $AdS_5 \times S^5$ form.

There is a question if we can trust the asymptotic solutions in the infrared. The problem is that when the coupling becomes strong, string corrections become important. The situation is different in the UV where we can trust our solutions because the coupling is small. The role of the α' corrections in the IR has been discussed in [14]. It was claimed that the α' corrections are small.

4. Cosmological evolution of the Brane-Universe

We consider a D3-brane moving along a geodesic in the background of a type 0 string. The metric on the brane (2.1) using the background solution (3.8) is

$$d\hat{s}^2 = (-H^{-\frac{1}{2}} + H^{\frac{1}{2}}\dot{r}^2 + H^{\frac{1}{2}}r^2 h_{ij}\dot{\varphi}^i \dot{\varphi}^j)dt^2 + H^{-\frac{1}{2}}(d\vec{x})^2 \quad (4.1)$$

The RR field $C = C_{0123}$ using the ansatz (3.9) becomes

$$C' = 2Qg^2 g_s^{-\frac{5}{2}} \sqrt{g_{rr}} f^{-1}(T) \quad (4.2)$$

where Q is a constant. Using again the solution (3.8) the RR field can be integrated to give

$$C = e^{-\Phi_0} f^{-1}(T) \left(1 + \frac{e^{\Phi_0} Q}{2r^4}\right)^{-1} + Q_1 \quad (4.3)$$

where Q_1 is a constant. The effective density on the brane (2.8), using eq.(3.8) and (4.2) becomes

$$\frac{8\pi}{3} \rho_{eff} = \frac{1}{4} [(f^{-1}(T) + EH e^{\Phi_0})^2 - (1 + \frac{\ell^2 e^{2\Phi_0}}{2} H)] \frac{Q^2 e^{2\Phi_0}}{r^{10}} H^{-\frac{5}{2}} \quad (4.4)$$

where the constant Q_1 was absorbed in a redefinition of the parameter E . Identifying $g = \alpha^2$ and using $g = H^{-\frac{1}{2}}$ we get from (4.4)

$$\begin{aligned} \frac{8\pi}{3}\rho_{eff} = & \left(\frac{2e^{-\Phi_0}}{Q}\right)^{\frac{1}{2}} \left[\left(f^{-1}(T) + \frac{Ee^{\Phi_0}}{\alpha^4}\right)^2 - \left(1 + \frac{\ell^2 e^{2\Phi_0}}{\alpha^6} \left(\frac{2e^{-\Phi_0}}{Q}\right)^{\frac{1}{2}} \right. \right. \\ & \left. \left. (1 - \alpha^4)^{\frac{1}{2}} \right) \right] (1 - \alpha^4)^{\frac{5}{2}} \end{aligned} \quad (4.5)$$

From the relation $g = H^{-\frac{1}{2}}$ we can see that the range of α is $0 \leq \alpha < 1$, while the range of r is $0 \leq r < \infty$. We can calculate the scalar curvature of the four-dimensional universe as

$$R_{brane} = 8\pi(4 + \alpha\partial_\alpha)\rho_{eff} \quad (4.6)$$

If we use the effective density of (4.5) it is easy to see that R_{brane} of (4.6) blows up at $\alpha = 0$. On the contrary if $r \rightarrow 0$, then the ds^2 of (2.1) becomes

$$ds_{10}^2 = \frac{r^2}{L}(-dt^2 + (d\vec{x})^2) + \frac{L}{r^2}dr^2 + Ld\Omega_5 \quad (4.7)$$

with $L = (\frac{e^{\Phi_0}Q}{2})^{\frac{1}{2}}$. This space is a regular $AdS \times S^5$ space.

Therefore the brane develops an initial singularity as it reaches $r = 0$, which is a coordinate singularity and otherwise a regular point of the AdS_5 space. This is another example in Mirage Cosmology [8] where we can understand the initial singularity as the point where the description of our theory breaks down.

If we take $\ell^2 = 0$, set the function $f(T)$ to its minimum value and also taking $\Phi_0 = 0$, the effective density (4.5) becomes

$$\frac{8\pi}{3}\rho_{eff} = \left(\frac{2}{Q}\right)^{\frac{1}{2}} \left(\left(2 + \frac{E}{\alpha^4}\right)^2 - 1 \right) (1 - \alpha^4)^{\frac{5}{2}} \quad (4.8)$$

As we can see in the above relation, there is a constant term, coming from the tachyon function $f(T)$. For small α and for some range of the parameters E and Q it gives an inflationary phase to the brane cosmological evolution. In Fig.1 we have plotted ρ_{eff} as a function of α for $Q = 2$.

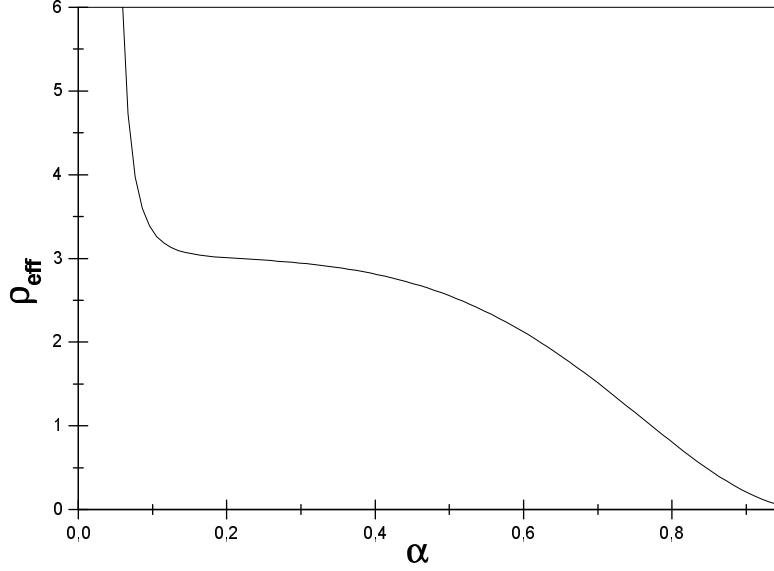


Figure 1: The induced energy density on the brane as a function of the brane scale factor.

As the brane moves away from $r = 0$ to larger values of r , the universe after the inflationary phase enters a radiation dominated epoch because the term α^{-4} takes over in (4.8). As the cosmic time η elapses the α^{-8} term dominates and finally when the brane is far away from $r = 0$, the term which is controlled by the angular momentum ℓ^2 gives the main contribution to the effective density. Non zero values of ℓ^2 will give negative values for ρ_{eff} . We expect that at later cosmic times there will be other fields, like gauge fields, which will give a different dynamics to the cosmological evolution and eventually cancel the negative matter density.

Let us now see what happens if the tachyon and dilaton fields are not constant. In the presence of a non trivial tachyon field the coupling $e^{-\Phi}$ which appears in the Dirac-Born-Infeld action in (2.2), is modified by a tachyonic function $\kappa(T) = 1 + \frac{1}{4}T + O(T^2)$. Then we can define an effective coupling [12]

$$e_{eff}^{-\Phi} = \kappa(T)e^{-\Phi} \quad (4.9)$$

The bulk fields are also coordinate dependent and the induced metric on the brane

will depend on a non trivial way on the dilaton field. Therefore the metric in the string frame will be connected to the metric in the Einstein frame through $g_{St} = e^{\frac{\Phi}{2}} g_E$. All the quantities used so far were defined in the string frame. We will follow our cosmological evolution in the Einstein frame. Then the relation (2.8) becomes

$$\frac{8\pi}{3}\rho_{eff} = \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{(C+E)^2 g_S - |g_{00}|(g_S g^3 + \ell^2)}{4|g_{00}|g_{rr}g_S g^3} \left(\frac{g'}{g}\right)^2 \quad (4.10)$$

Having the approximate solution in the UV given by (3.15)-(3.18) we can calculate the metric components of the metric (2.1) and find

$$g_{yy} = \frac{1}{16} \sqrt{\frac{Q}{2}} \left(1 - \frac{9}{2y}\right), g_s = \sqrt{\frac{Q}{2}} \left(1 - \frac{1}{2y}\right) \quad (4.11)$$

$$g = \frac{1}{\sqrt{2Q}} e^{\frac{y}{2}} \left(1 - \frac{1}{2y}\right) \quad (4.12)$$

The variable y is defined by $\rho = e^{-y}$. Then we can identify g of (4.12) with the scale factor α^2 and solve for y . We get two solutions

$$y_1 = -\frac{1}{4\log\alpha + \log 2Q}, y_2 = \log 2Q + 4\log\alpha + \frac{1}{\log 2Q + 4\log\alpha} \quad (4.13)$$

From the solution y_2 which has the right behaviour for large α , we keep the $\log 2Q + 4\log\alpha$ term. Then the RR field C becomes

$$C = \frac{e^y}{Q} - \frac{2}{Q} Ei[y] \quad (4.14)$$

Then, we can calculate the effective energy density from (4.10) setting $\ell^2 = 0$ and we get

$$\begin{aligned} \frac{8\pi}{3}\rho_{eff} &= \left[\left(1 - \frac{1}{Q\alpha^4} Ei[\log 2Q + 4\log\alpha] + \frac{E}{2\alpha^4}\right)^2 - \frac{1}{4} \left(1 - \frac{1}{2(\log 2Q + 4\log\alpha)}\right)^4 \right. \\ &\quad \left. \left(1 - \frac{1}{2(\log 2Q + 4\log\alpha)}\right)^{-4} \left(1 - \frac{9}{2(\log 2Q + 4\log\alpha)}\right)^{-1} \right. \\ &\quad \left. \left(1 + \frac{1}{(\log 2Q + 4\log\alpha)^2} \frac{1}{\left(1 - \frac{1}{2(\log 2Q + 4\log\alpha)}\right)}\right)^2 \right] \end{aligned} \quad (4.15)$$

For some typical value of the parameters $Q=1$ and $E=1$, and for large values of α , it is obvious that ρ_{eff} has a constant value. Therefore an observer on the brane will see an expanding inflating universe. It is interesting to see what happens for small values of α . As α gets smaller, a term proportional to $\frac{1}{(\log \alpha)^4}$ starts to contribute to ρ_{eff} . Therefore the universe for small values of scale factor has a slow expanding inflationary phase. For smaller value of α we cannot trust the solution which is reflected in the fact that ρ_{eff} gets infinite. The behaviour of the effective energy density as a function of the scale factor is shown in Fig.2

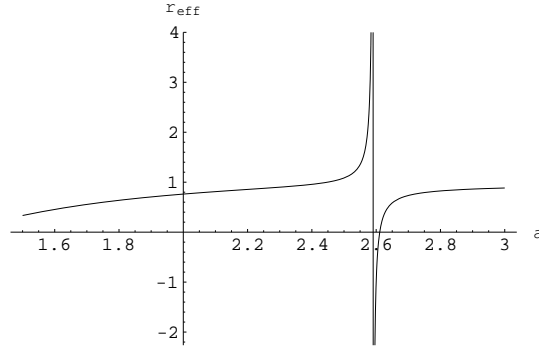


Figure 2: The induced energy density on the brane as a function of the brane scale factor.

Going now to IR using (3.20)-(3.23) we get for the metric components

$$g_{yy} = \frac{\sqrt{Q}}{16} 2^{-\frac{3}{4}} \left(1 - \frac{1}{2y}\right), g_s = 2^{-\frac{3}{4}} \sqrt{Q} \left(1 + \frac{7}{2y}\right) \quad (4.16)$$

$$g = \frac{2^{-\frac{1}{4}}}{\sqrt{Q}} e^{-\frac{y}{2}} \left(1 - \frac{9}{2y}\right) \quad (4.17)$$

where now y is defined by $\rho = e^y$.

Then the identification $g = \alpha^2$ using (4.17) gives again two solutions

$$y_1 = -\frac{9}{4\log \alpha + \log \sqrt{2}Q}, y_2 = -\log \sqrt{2}Q - 4\log \alpha + \frac{9}{\log \sqrt{2}Q + 4\log \alpha} \quad (4.18)$$

For small α we keep from the solution y_2 the term $-\log \sqrt{2}Q - 4\log \alpha$. Using this solution we can calculate the RR field

$$C = -\frac{e^{-y}}{Q} - \frac{2}{Q} Ei[-y] \quad (4.19)$$

Then ρ_{eff} becomes,

$$\begin{aligned}
\frac{8\pi}{3}\rho_{eff} = & \left[\left(-1 - 2\frac{1}{\sqrt{2}Q\alpha^4} Ei[\log\sqrt{2}Q + 4\log\alpha] + \frac{E}{\sqrt{2}\alpha^4} \right)^2 - \right. \\
& \left. \frac{1}{2} \left(1 + \frac{9}{2(\log\sqrt{2}Q + 4\log\alpha)} \right)^4 \right] \left(1 + \frac{9}{2(\log\sqrt{2}Q + 4\log\alpha)} \right)^{-4} \\
& \left(1 + \frac{1}{2(\log\sqrt{2}Q + 4\log\alpha)} \right)^{-1} \\
& \left(1 - \frac{9}{2(4\log\alpha + \log\sqrt{2}Q)^2} \frac{1}{\left(1 - \frac{9}{2(\log\sqrt{2}Q + 4\log\alpha)} \right)} \right)^2
\end{aligned} \tag{4.20}$$

As we can see, the above relation is the same as the energy density in the UV (relation (4.15)) up to some numerical factors, as expected. The difference is, that now it is valid for small α . For small α first the term $\frac{1}{\alpha^8}$ dominates and then the term $\frac{1}{\alpha^4}$. As α increases the term $\frac{1}{(\log\alpha)^4}$ takes over and drives the universe to a slow inflationary expansion.

5. Discussion

We had followed a probe brane along a geodesic in the background of type 0 string. Assuming that the universe is described by a three-dimensional brane, we calculate the effective energy density which is induced on the brane because of this motion. We studied this mirage matter as the brane-universe moves along the radial coordinate.

Using an exact solution of the type 0 string theory, we found [9] that the motion of the brane-universe in this particular background induces an inflationary phase on the brane. In the case when the dilaton and tachyon fields are coordinate dependent, there are approximate solutions for large values of the radial coordinate, in the UV region and solutions for small values of the radial coordinate in the IR. In the UV the effective coupling of the theory is small, so we can trust the approximate solutions. In the IR, the effective coupling becomes strong but it was shown in the literature [14], that all string corrections are small.

Using these solutions, we calculated the energy densities that are induced on the brane. What we found is that for large values of the scale factor as it is measured on the brane (large values of the radial coordinate) the universe enters a slow inflationary phase, in which the energy density is proportional to an inverse power of the logarithm of the scale factor. As the scale factor grows the induced energy density takes a constant value and the universe enters a normal exponential expansion. For small values of the scale factor the induced energy density scales as the inverse powers of the scale factor and then the logarithmic terms take over and the universe enters a slow exponential expansion.

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